2011 ECTC Proceedings
ASME Early Career Technical Conference
Hosted by ASME District E and University of Arkansas
Support Provided by the ASME Old Guard and
the Committee on Early Career Development
March 31-April 02, 2011, Fayetteville, AR

REVISION OF B31 CODE EQUATIONS FOR STRESS INTENSIFICATION FACTORS AND FLEXIBILITY FACTORS FOR INTERSECTIONS

Peter Q. Vu
Paulin Research Group
Houston, TX 77082

ABSTRACT

ASME Standards and Technology, LLC (ASME ST-LLC) awarded Paulin Research Group (PRG) a project to align stress intensification factors (SIFs) and flexibility factors for piping components. Finite element analysis was performed for various sizes and types of pipe intersection geometries. Nonlinear regression was used to produce correlation equations from the FEA data in accordance with new code procedures established by E. Rodabaugh in EPRI TR-110996. The existing B31 SIFs and flexibility factors were compiled and compared to PRG FEA and test data through the use of 2D and 3D graphical representations, which allow clear, visual interpretation of the intersection's geometric parameters. The correlation equations for SIFs and flexibility factors may provide more applicable data for the design of piping systems.

INTRODUCTION

There has been discussion regarding the accuracy of the ASME B31 Code equations used to calculate stress intensification factors and flexibility factors for branch connections in piping systems. SIFs allow the piping designer estimates of the fatigue performance of particular piping components [1]. Early fatigue based SIFs were first developed in the early 1950s by Markl [2]. Considerable research was done in the years following, and many papers were published concerning SIFs and flexibility factors. Key factors for determining SIFs include load deflection, cyclic bending tests, defining failures, and SIF determination [3]. Design rules for SIFs and flexibility factors were established and published as a part of the B31 Code. With more and more research being done, many have proposed revisions to the Code.

Finite element analysis is a method many piping stress analysts use to study SIFs and flexibility factors. FEA technology is well understood and reliable. SIFs and flexibilities are based on analytical and empirical relations correlated to piping component geometries and can be calculated for all geometries and load conditions. These calculations can then be compiled and analyzed.

Paulin Research Group has looked into developing newer design criteria for SIFs and flexibility factors for piping intersections. The remainder of this text is devoted to presenting the work that has been done on my end towards this on-going project.

NOMENCLATURE

D	mean diameter of run pipe, in.
D_o	outside diameter of run pipe, in.
d	mean diameter of branch pipe, in.
d_o	outside diameter of branch pipe, in.
i	stress intensification factor, SIF
k	flexibility factor
M	bending moment, in.-lb
R	mean radius of run pipe, in.
T	nominal wall thickness of run pipe, in.
t	nominal wall thickness of branch pipe, in.
Z	section modulus of pipe, in.^{3}

PROCEDURE

Database Point Plotter. Paulin Research Group had compiled a large database of finite element analysis runs of pipe intersections of varying sizes. The main geometric parameters looked at are d/D, D/T, and t/T. They are the ratios of the mean diameter of the nozzle to the mean diameter of the vessel, the mean diameter of the vessel to the thickness of the vessel, and the thickness of the nozzle to the thickness of the vessel. The geometric parameters of the large database can be seen in Table 1. Industry standards usually don't use such a large range. To be more in line, PRG only uses a portion of the range, which can be seen in Table 2, identified as the Quality Control (QC) Database.

Table 1: Geometric Parameters of PRG Database

	d/D	D/T	t/T
Min	0.05	8	0.0029
Max	1	140	14.875

Table 2: Geometric Parameters of PRG QC Database

	d/D	D/T	t/T
Min	0.1	8	0.3
Max	1	100	3

I modified an Excel-based program using Visual Basic for Applications (VBA) to call on a 3D plotter program that PRG created. Code was written that read in thousands of d/D, D/T, t/T, SIF, and flexibility data. This data was sorted and code was written out to plot points with D/T on the X axis, SIF or flexibility factor on the Y axis, d/D on the Z axis, and t/T in the fourth dimension, by color coding it. An example of the output from the 3D plotter can be seen in Figure 1. Axis labeling was changed to follow a more conventional system used in process piping. The axis are labeled R/T, r/R, and SCF, where SCF is the stress concentration factor. SCF is equal to twice the SIF divided by 1.35. The radius r and R replace diameter values d and D respectively.

The models from the QC database could now be plotted in 3D and viewed comparatively. Older methods of comparing SIFs were to plot several equations in 2D space on paper. With the 3D plotter, we are able to compare the relationship of the SIF or flexibility factor to its individual geometric components.

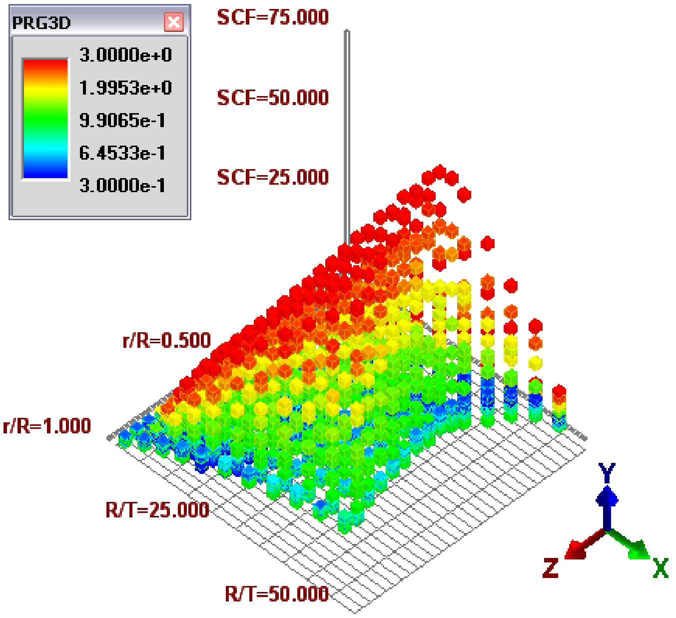
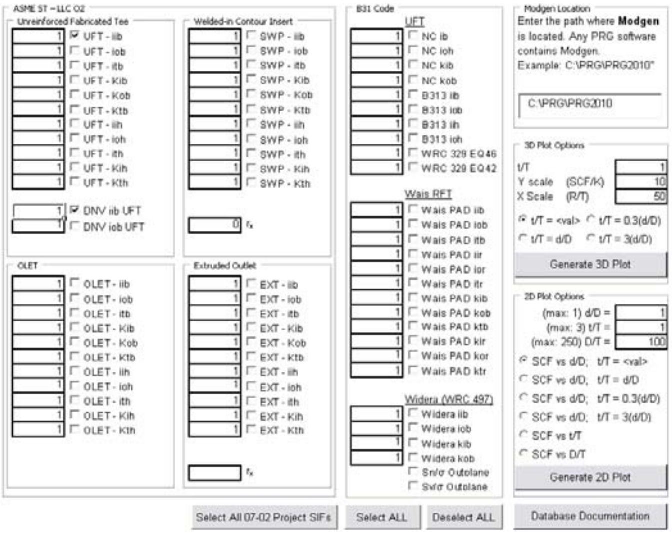


Figure 1: 3D Plotter Output

Nonlinear Regression. After making FEA runs of the thousands of pipe intersection models and plotting them, there needed to be a method to correlate the dataset. Nonlinear regression on the dataset would be used to produce correlation equations for different loading cases for SIFs and flexibility factors. NLREG, a statistical software program written by Phillip H. Sherrod, was used. STATISTICA, another statistical program, was used to verify our results. PRG decided to use NLREG because there was more documentation and feedback given from Mr. Sherrod.

The results of the nonlinear regression were incorporated into the 3D plotter program. The correlation equations were used to generate surface plots for six different loading cases for SIFs and flexibility factors, making a total of 12 correlation equations for each type of intersection. The six intersections of interest are welding tees, pad reinforced fabricated tees, unreinforced fabricated tees, extruded outlets, welded-in contour inserts, and integrally reinforced branch welded-on fittings. The six loading cases of interests are inplane bending with a moment applied to the branch, out-of-plane bending with a moment applied to the branch, torsion with a moment applied to the branch, inplane bending with a moment applied to the vessel, out-of-plane bending with a moment applied to the vessel, and torsion with a moment applied to the vessel. They are represented as iib, iob, itb, iih, ioh, and ith, respectively. The first letter, "i" represents a SIF. The second letter indicates either inplane, out-of-plane, or torsional loading. The third letter indicates whether the loading is applied to the branch of the header (vessel). Correlation equations for flexibility factors were also made. They are represented in the same way as SIFs. Flexibility factors are kib, kob, ktb, kih, koh, and kth.

Plot Comparison. With surface plots of these load cases available, we then needed to compare these SIFs and flexibility factors with industry standards. Modification of the plotter program was done to include B31, NC, Widera, and Wais SIFs and flexibility factors [4, 5, 6, 7]. An option to plot in 2D with Excel charts was also included. A checkbox system was employed to allow the user the ability to select which surfaces or lines to be compared. This allowed for a comprehensive comparison of SIFs and flexibility factors, and we could then identify where there could be better relationships. Figure 2 shows a condensed interface for the 3D and 2D plotter tool. The actual tool includes more intersection types and more B31 Code equations, which couldn't be included in the figure due to its large size. The left section consists or proposed correlation equations. The middle section consists of B31, NC, Widera, and Wais equations. The right section consists of 3D and 2D plotting controls. The plot options allow the user to plot SIFs and flexibility factors against individual geometric parameters.



ASME ST – LLC 02

Unreinforced Fabricated Tee
1 ☑ UFT - iib
1 ☐ UFT - iob
1 ☐ UFT - itb
1 ☐ UFT - Kib
1 ☐ UFT - Kob
1 ☐ UFT - Ktb
1 ☐ UFT - iih
1 ☐ UFT - ioh
1 ☐ UFT - ith
1 ☐ UFT - Kih
1 ☐ UFT - Kth
1 ☑ DNV iib UFT
1 ☐ DNV iob UFT

Welded-in Contour Insert
1 ☐ SWP - iib
1 ☐ SWP - iob
1 ☐ SWP - itb
1 ☐ SWP - Kib
1 ☐ SWP - Kob
1 ☐ SWP - Ktb
1 ☐ SWP - iih
1 ☐ SWP - ioh
1 ☐ SWP - ith
1 ☐ SWP - Kih
1 ☐ SWP - Kth
0 fx

OLET
1 ☐ OLET - iib
1 ☐ OLET - iob
1 ☐ OLET - itb
1 ☐ OLET - Kib
1 ☐ OLET - Kob
1 ☐ OLET - Ktb
1 ☐ OLET - iih
1 ☐ OLET - ioh
1 ☐ OLET - ith
1 ☐ OLET - Kih
1 ☐ OLET - Kth

Extruded Outlet
1 ☐ EXT - iib
1 ☐ EXT - iob
1 ☐ EXT - itb
1 ☐ EXT - Kib
1 ☐ EXT - Kob
1 ☐ EXT - Ktb
1 ☐ EXT - iih
1 ☐ EXT - ioh
1 ☐ EXT - ith
1 ☐ EXT - Kih
1 ☐ EXT - Kth
fx

B31 Code

UFT
1 ☐ NC ib
1 ☐ NC ioh
1 ☐ NC kib
1 ☐ NC kob
1 ☐ B313 ib
1 ☐ B313 iob
1 ☐ B313 ih
1 ☐ B313 ioh
1 ☐ WRC 329 EQ46
1 ☐ WRC 329 EQ42

Wais RFT
1 ☐ Wais PAD iib
1 ☐ Wais PAD iob
1 ☐ Wais PAD itb
1 ☐ Wais PAD iir
1 ☐ Wais PAD ior
1 ☐ Wais PAD itr
1 ☐ Wais PAD kib
1 ☐ Wais PAD kob
1 ☐ Wais PAD ktb
1 ☐ Wais PAD kir
1 ☐ Wais PAD kor
1 ☐ Wais PAD ktr

Widera (WRC 497)
1 ☐ Widera iib
1 ☐ Widera iob
1 ☐ Widera kib
1 ☐ Widera kob
☐ Sn/σ Outplane
☐ Sv/σ Outplane

Modgen Location

Enter the path where **Modgen** is located. Any PRG software contains Modgen. Example: C:\PRG\PRG2010"

C:\PRG\PRG2010

3D Plot Options
t/T 1
Y scale (SCF/K) 10
X Scale (R/T) 50
● t/T = <val> ○ t/T = 0.3(d/D)
○ t/T = d/D ○ t/T = 3(d/D)

2D Plot Options
(max: 1) d/D = 1
(max: 3) t/T = 1
(max: 250) D/T = 100
● SCF vs d/D; t/T = <val>
○ SCF vs d/D; t/T = d/D
○ SCF vs d/D; t/T = 0.3(d/D)
○ SCF vs d/D; t/T = 3(d/D)
○ SCF vs t/T
○ SCF vs D/T

Figure 2: Condensed Plotter Interface

RESULTS AND DISCUSSION

The six different loading cases for unreinforced fabricated tees are shown below with PRG proposed equations for SIFs and flexibility factors. Figures 3 through 14 show 2D comparison plots with B31 Code equations for their respective load cases. SIFs or flexibility factors are plotted against d/D. D/T is equal to 100, and t/T is equal to 1.

SIF: Inplane Moment on the Branch. Widera provides two separate equations for nozzle SIFs and vessel SIFs [1]. The larger of the two is used to plot the SIF. For a t/T ratio of 1, Widera's SIFs are larger in the vessel than the nozzle. For the inplane case, Widera's SIFs are lower than the current B31 code, Wais, and PRG proposed SIFs. PRG lies more in line with Wais and is more conservative than the current B31 Code equations.

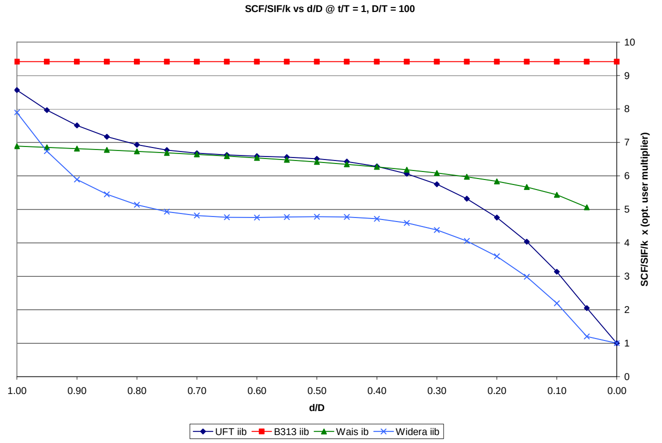


Figure 3: Comparison Plot for iib

$$iib = \left[0.038 + 1.45\left(\frac{d}{D}\right) - 2.39\left(\frac{d}{D}\right)^2 + 1.34\left(\frac{d}{D}\right)^3\right]\left(\frac{R}{T}\right)^{0.76}\left(\frac{t}{T}\right)^{0.74} \quad (1)$$

SIF: Out-of-plane Moment on the Branch. For the out-of-plane load case, PRG lies more in line with Widera than Wais. PRG recommended SIFs adequately model the geometries without being too conservative.

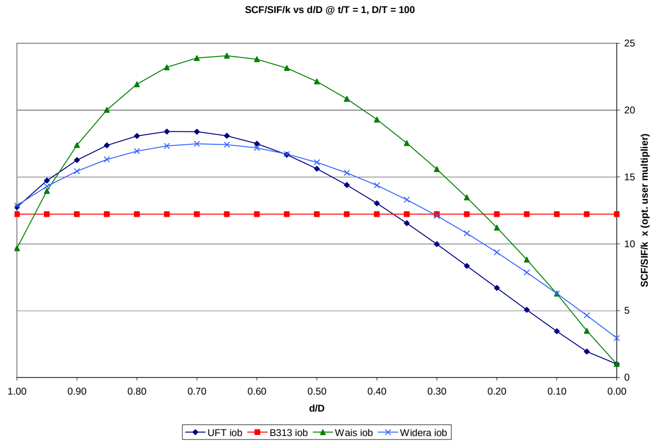


Figure 4: Comparison Plot for iob

$$iob = \left[0.038 + 2\left(\frac{d}{D}\right) + 2\left(\frac{d}{D}\right)^2 - 3.1\left(\frac{d}{D}\right)^3\right]\left(\frac{R}{T}\right)^{2/3}\left(\frac{t}{T}\right) \quad (2)$$

SIF: Torsional Moment on the Branch. B31 Code does not provide an equation for torsional SIFs, nor does Widera. The only SIFs used for comparison are from Wais.

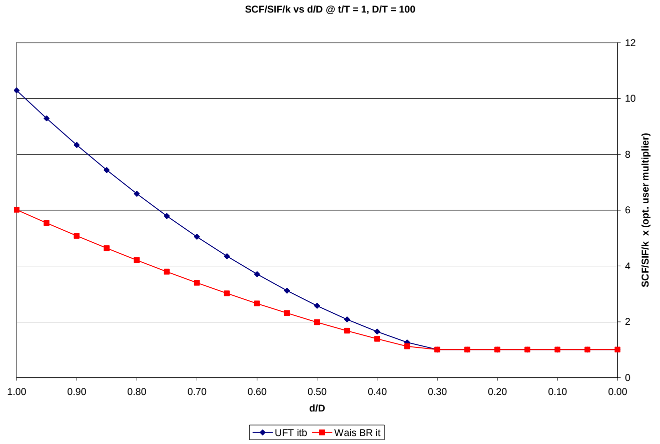


Figure 5: Comparison Plot for itb

$$itb = 0.45\left(\frac{R}{T}\right)^{0.8}\left(\frac{t}{T}\right)^{0.29}\left(\frac{d}{D}\right)^2 \quad (3)$$

SIF: Inplane Moment on the Header. PRG recommended SIFs for moments applied to the header are conservative compared to the older B31 design criteria. This behavior is also seen in the out-of-plane header case as well.

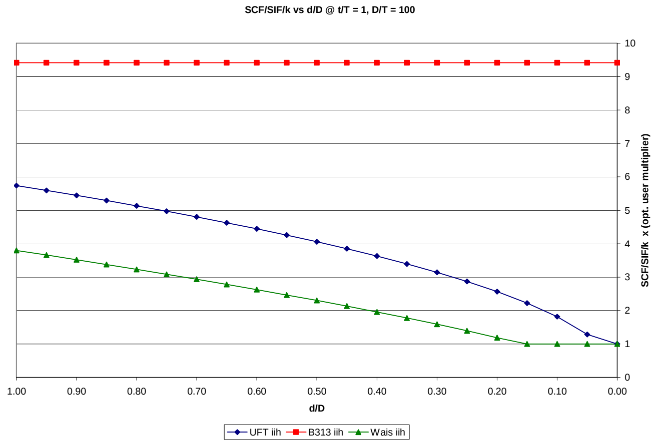


Figure 6: Comparison Plot for iih

$$iih = 1.2\left(\frac{d}{D}\right)^{0.5}\left(\frac{R}{T}\right)^{0.4}\left(\frac{t}{T}\right)^{-0.35} \geq 1.5 \quad (4)$$

SIF: Out-of-plane Moment on the Header.

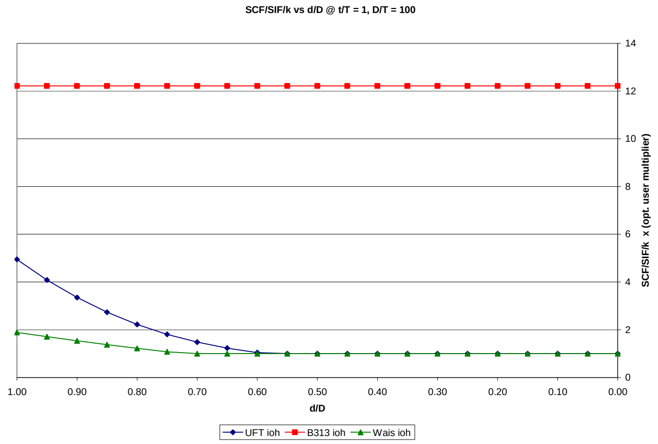


Figure 7: Comparison Plot for ioh

$$ioh = \left[\left(\frac{d}{D}\right) - 2.7\left(\frac{d}{D}\right)^2 + 2.62\left(\frac{d}{D}\right)^3\right]\left(\frac{R}{T}\right)^{0.43}\left(\frac{t}{T}\right)^{-0.7} \qquad (5)$$

SIF: Torsional Moment on the Header.

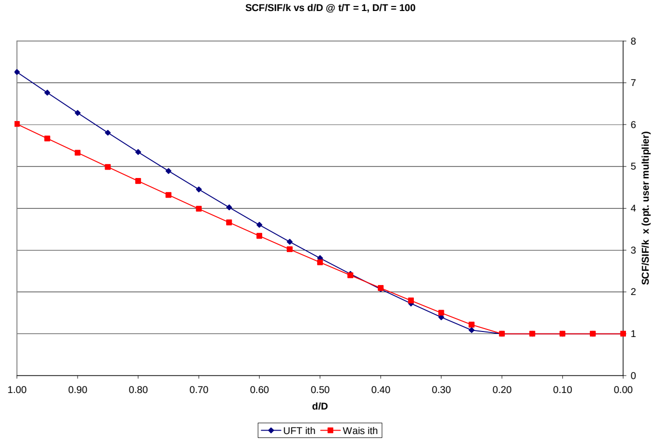


Figure 8: Comparison Plot for ith

$$ith = 1.2\left(\frac{R}{T}\right)^{0.46}\left(\frac{t}{T}\right)^{-0.45}\left(\frac{d}{D}\right)^{1.37} \qquad (6)$$

Flexibility Factor: Inplane Moment on the Branch. The PRG recommended flexibility factor for inplane moment applied on the branch matched very well with comparable data. It lies within range of Widera, Wais, and B31 code flexibilities.

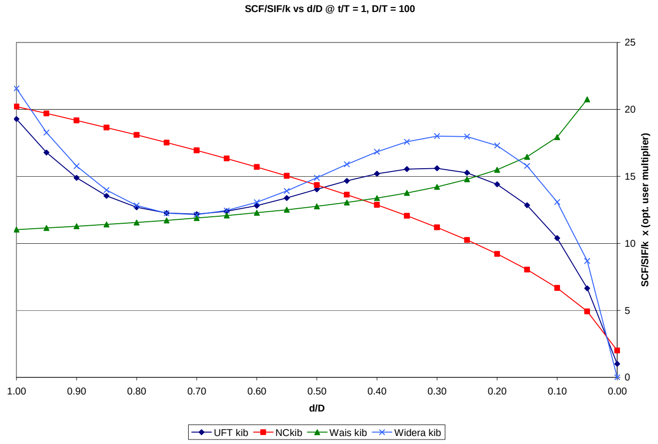


Figure 9: Comparison Plot for kib

$$kib = \left[3.15\left(\frac{d}{D}\right) - 6.4\left(\frac{d}{D}\right)^2 + 4\left(\frac{d}{D}\right)^3\right]\left(\frac{R}{T}\right)^{0.83}\left(\frac{t}{T}\right)^{0.49}\left(\frac{d}{D}\right)^{-0.2} \qquad (7)$$

Flexibility Factor: Out-of-plane Moment on the Branch.

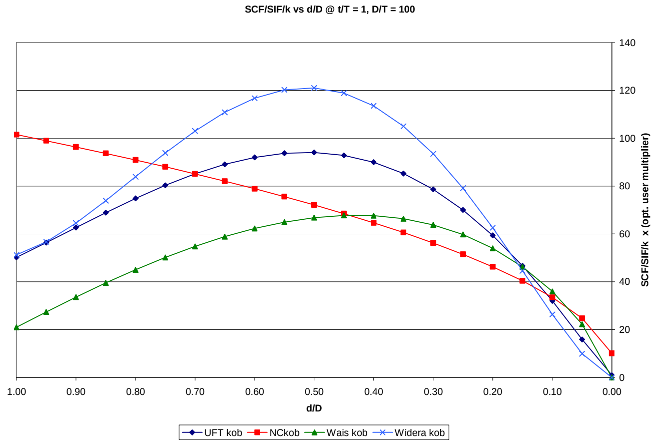


Figure 10: Comparison Plot for kob

$$kob = \left[2.05\left(\frac{d}{D}\right) - 2.94\left(\frac{d}{D}\right)^2 + 1.1\left(\frac{d}{D}\right)^3\right]\left(\frac{R}{T}\right)^{1.4}\left(\frac{t}{T}\right)^{0.6}\left(\frac{d}{D}\right)^{0.12} \qquad (8)$$

Flexibility Factor: Torsional Moment on the Branch. B31 equations are not supplied for this particular case. A comparison to the loading done by Wais shows that the PRG recommended flexibility factor matches very well.

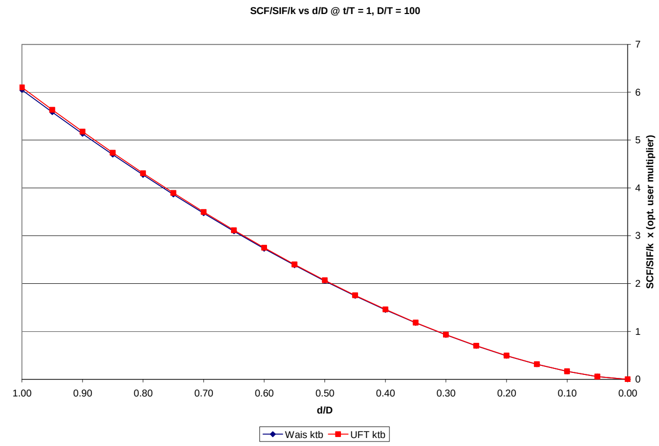


Figure 11: Comparison Plot for ktb

$$ktb = 2.79\left(\frac{R}{T}\right)^{0.2}\left(\frac{t}{T}\right)^{0.55}\left(\frac{d}{D}\right)^{1.56} \qquad (9)$$

Flexibility Factor: Inplane Moment on the Header.

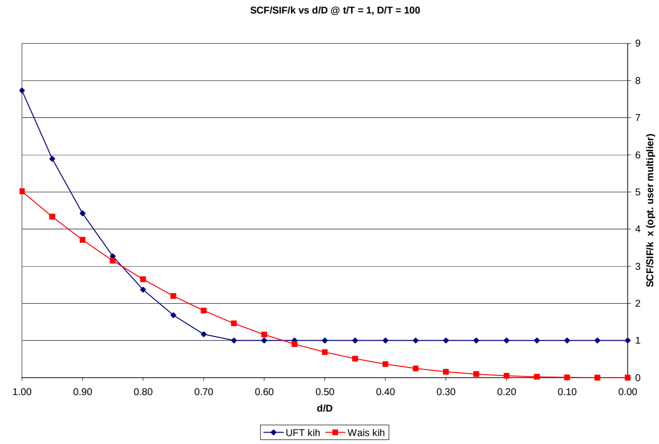


Figure 12: Comparison Plot for kih

$$kih = 1.23\left(\frac{R}{T}\right)^{0.47}\left(\frac{t}{T}\right)^{-0.47}\left(\frac{d}{D}\right)^{5.3} \qquad (10)$$

Flexibility Factor: Out-of-plane Moment on the Header.

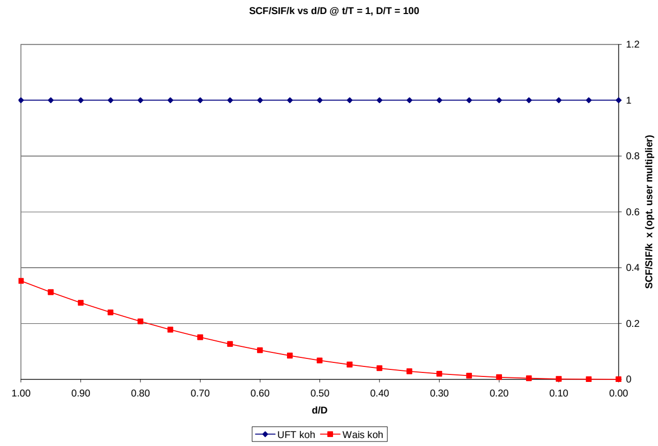


Figure 13: Comparison Plot for koh

$$koh = 1 \qquad (11)$$

Flexibility Factor: Torsional Moment on the Header.

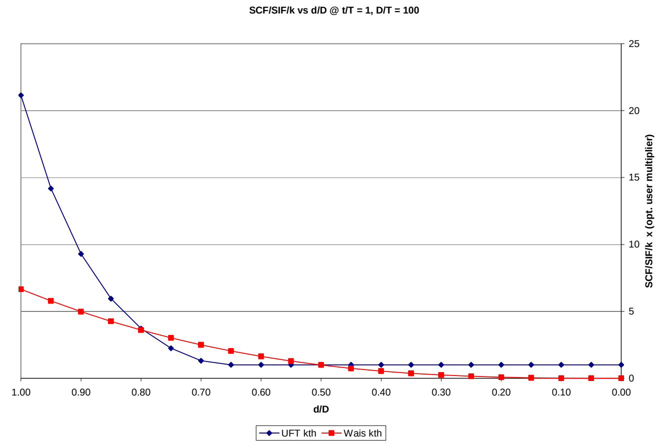


Figure 14: Comparison Plot for kth

$$kth = \left(\frac{R}{T}\right)^{0.78}\left(\frac{t}{T}\right)^{-0.8}\left(\frac{d}{D}\right)^{7.8} \qquad (12)$$

See Appendix A for complete list of SIF and flexibility factor correlation equations for the six intersection types discussed.

CONCLUSION

Recommended SIF and flexibility factors can have significant affects on piping system analysis. Five example models were prepared as part of the ASME project. The results of these models are summarized in Table 3.

Table 3: Results of Example Models

Example No.	Description	Result
1	Rodabaugh Flexibility Example from WRC 329 Fig. 15	Branch connection flexibilities reduce moment by 8.89 times.

2	4x4 Markl Unreinforced Fabricated Tee Piping Assembly	Without branch connection flexibilities displacement underestimated by 22%.
3	Piping Attached to Pump Discharge	Including branch connection flexibilities reduces pump flange moment by 43.7%.
4	Spare Pump Branch Configuration	Branch and run flexibilities reduce moment by 3.2 times.
5	Heater Piping	Branch and run flexibilities increase moment by 2.4 times.

Figure 15 shows model number 1. Everett Rodabaugh provides the following example in Section 4.9 of WRC 329 [8].

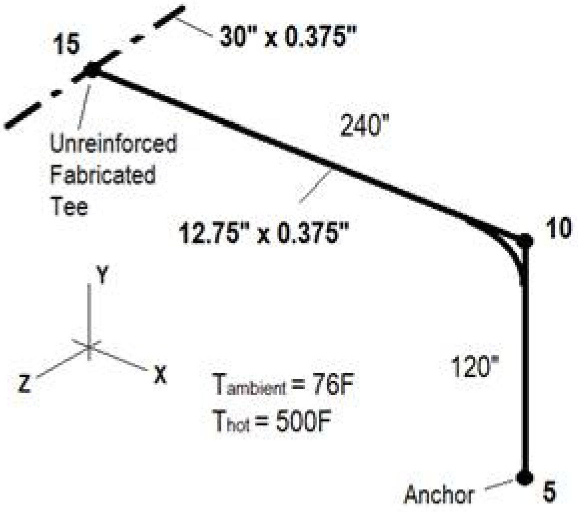


Figure 15: WRC 329 Fig. 15 Example Piping Model

Without considering the branch connection flexibility of the 12x30" fabricated tee at point 15 the out-of-plane (Z) bending moment at point 15 is 372,000 in.lb. Including the branch connection flexibility reduces the bending moment to 41,832 in.lb., a reduction of 8.8.

B31 SIFs are easily determined and applied. They've been around for quite some time. Piping designers can rely on them because the SIFs have been validated for years. As seen above in Table 3, the correlation equations for SIFs and flexibility factors can affect the piping system considerably. It is in this hope that PRG recommended SIFs and flexibility factors can provide more applicable data for piping system design.

ACKNOWLEDGMENTS

I would like to thank Phillip H. Sherrod for the development of the Nonlinear Regression Analysis program and providing Paulin Research Group with a copy of the software. I would also like to thank Willy Lock and Dominic Jordan for their assistance with VBA coding. I would also like to thank Anthony Paulin for his continued support and guidance throughout this research.

REFERENCES

1) http://www.sstusa.com/pipe-stress-article-piping-sif.php , Haupt, Ron, Stress Intensification Factors, February 3, 2011.

2) Williams, D. K., 1995, "Development of Stress Intensification Factors for Fabricated Tee Branch Connections," Developments in Pressure Vessel Piping, Vol. 301, 1995.

3) Rodabaugh, E.C., "Standardized Method for Developing Stress Intensification Factors for Piping Components," WRC Bulletin 392, Welding Research Council, New York, N.Y., 1994.

4) WRC 497 Large Diameter Ratio Shell Intersections, Koves, Mokhtarian, Rodabaugh, Widera, December 2004. Widera – WRC 497 Part 3: Parametric Finite Element Analysis of Large Diameter Shell Intersections (External Loadings) G.E.O. Widera and Z. Wei. "Large Diameter Ratio Shell Intersections", WRC 497 Parts 1,2 and 3, by G.E.O. Widera and Z. Wei

5) E. Wais, Rodabaugh, E.C., "Stress Intensification Factors and Flexibility Factors for Unreinforced Branch Connections", TR 110996 Final Report, Issued November 1998.

6) ASME, 2006, ASME Code for Pressure Piping B31, "Process Piping", 2006 Edition, The American Society of Mechanical Engineers, New York, N.Y.

7) Xue, L., Widera,G.,E.,O., Sang, Z., "Flexibility Factors for Branch Pipe Connections Subjected to In-Plane and Out-of-Plane Moments", Journal of Pressure Vessel Technology, Feb.,2006, Vol. 128.

8) Rodabaugh, E.C., "Accuracy of Stress Intensification Factors for Branch Connections," WRC Bulletin 329, Welding Research Council, New York, N.Y., 1987.

APPENDIX A

*NOTE: “Run” is equivalent to “Header” and “Vessel.”

Welding tee per ASME B16.9	
Term	**Equation**
Run Inplane Flexibility Factor, k_{ih}	$0.18\ (R/T)^{0.91}\ (d/D)^{5}$
Run Outplane Flexibility Factor, k_{oh}	1
Run Torsional Flexibility Factor, k_{th}	$0.08\ (R/T)^{0.91}\ (d/D)^{5.7}$
Branch Inplane Flexibility Factor, k_{ib}	$(1.91(d/D) - 4.32(d/D)^2 + 2.7(d/D)^3)\ (R/T)^{0.77}\ (d/D)^{0.47}(t/T)$
Branch Outplane Flexibility Factor, k_{ob}	$(0.34(d/D) - 0.49(d/D)^2 + 0.18(d/D)^3)\ (R/T)^{1.46}(t/T)$
Branch Torsional Flexibility Factor, k_{tb}	$(1.08(d/D) - 2.44(d/D)^2 + 1.52(d/D)^3)\ (R/T)^{0.77}\ (d/D)^{1.61}(t/T)$
Run SIF Inplane, i_{ih}	$0.34\ (R/T)^{2/3}\ (d/D)^{1.98}$
Run SIF Outplane, i_{oh}	$0.29\ (R/T)^{2/3}\ (d/D)^{2.69}$
Run SIF Torsional, i_{th}	$0.34\ (R/T)^{2/3}\ (d/D)^{2.69}$
Branch SIF Inplane i_{ib}	$0.33\ (R/T)^{2/3}\ (d/D)^{0.86}$
Branch SIF Outplane, i_{ob}	$0.42\ (R/T)^{2/3}\ (d/D)^{0.75}$
Branch SIF Torsional, i_{tb}	$0.42\ (R/T)^{2/3}\ (d/D)^{2}$

Reinforced fabricated tee	(when t_p>1.5T use t_p=1.5T)
Term	**Equation**
Run Inplane Flexibility Factor, k_{ih}	$0.21\ (R / [T+0.5t_p]\ )^{0.97}\ (t/T)^{-0.65}\ (d/D)^{6.2}$
Run Outplane Flexibility Factor, k_{oh}	1
Run Torsional Flexibility Factor, k_{th}	$0.12\ (R / [T+0.5t_p]\ )^{1.39}\ (t/T)^{-0.74}\ (d/D)^{8.5}$
Branch Inplane Flexibility Factor, k_{ib}	$(1.29(d/D) - 2.73(d/D)^2 + 1.62(d/D)^3\ )\ (R / [T+0.5t_p]\ )^{1.2}\ (t/T)^{0.56}\ (d/D)^{0.33}$
Branch Outplane Flexibility Factor, k_{ob}	$(0.84(d/D) - 1.27(d/D)^2 + 0.5(d/D)^3)\ (R / [T+0.5t_p]\ )^{1.69}\ (t/T)^{0.68}\ (d/D)^{0.21}$
Branch Torsional Flexibility Factor, k_{tb}	$2.79\ (R / [T+0.5t_p]\ )^{0.75}\ (t/T)^{0.55}\ (d/D)^{1.56}(R/T)^{-0.55}$
Run SIF Inplane, i_{ih}	$(1.9(d/D) - 3.6(d/D)^2 + 2.53(d/D)^3\ )\ (R / [T+0.5t_p]\ )^{0.35}\ (t/T)^{-0.34}$
Run SIF Outplane, i_{oh}	$(1.29(d/D) - 2.87(d/D)^2 + 2.39(d/D)^3\ )\ (R / [T+0.5t_p]\ )^{0.35}$
Run SIF Torsional, i_{th}	$0.6\ (R / [T+0.5t_p]\ )^{2/3}\ (t/T)^{-0.69}\ (d/D)^{2}$
Branch SIF Inplane i_{ib}	$(3.33(d/D) - 5.49(d/D)^2 + 2.94(d/D)^3)\ (TR^{2/3})\ (T+0.5t_p)^{-5/3}\ (t/T)^{0.3}$
Branch SIF Outplane, i_{ob}	$(2.86(d/D) - 1.48(d/D)^2 - 0.48(d/D)^3)\ (TR^{2/3})\ (T+0.5t_p)^{-5/3}\ (t/T)$ (when $t/T < 1$ use $t/T=1$)
Branch SIF Torsional, i_{tb}	$1.07\ (d/D)^2\ (TR^{2/3})\ (T+0.5t_p\ )^{-5/3}\ (t/T)^{0.3}$

Unreinforced fabricated tee	
Term	**Equation**
Run Inplane Flexibility Factor, k_{ih}	$1.23\ (R/T)^{0.47}\ (t/T)^{-0.47}\ (d/D)^{5.3}$
Run Outplane Flexibility Factor, k_{oh}	1
Run Torsional Flexibility Factor, k_{th}	$(R/T)^{0.78}\ (t/T)^{-0.8}(d/D)^{7.8}$
Branch Inplane Flexibility Factor, k_{ib}	$(3.15(d/D) - 6.4(d/D)^2 + 4(d/D)^3)\ (R/T)^{0.83}\ (t/T)^{0.49}\ (d/D)^{-0.2}$
Branch Outplane Flexibility Factor, k_{ob}	$(2.05(d/D) - 2.94(d/D)^2 + 1.1(d/D)^3)\ (R/T)^{1.4}\ (t/T)^{0.6}(d/D)^{0.12}$
Branch Torsional Flexibility Factor, k_{tb}	$2.79\ (R/T)^{0.2}(t/T)^{0.55}(d/D)^{1.56}$
Run SIF Inplane, i_{ih}	$1.2(d/D)^{0.5}(R/T)^{0.4}\ (t/T)^{-0.35}$
Run SIF Outplane, i_{oh}	$((d/D) - 2.7(d/D)^2 + 2.62(d/D)^3)\ (R/T)^{0.43}\ (t/T)^{-0.7}$
Run SIF Torsional, i_{th}	$1.2(R/T)^{0.46}\ (t/T)^{-0.45}(d/D)^{1.37}$ (when $t/T < 0.15$ use $t/T = 0.15$)
Branch SIF Inplane i_{ib}	$(0.038 + 1.45(d/D) - 2.39(d/D)^2 + 1.34(d/D)^3)\ (R/T)^{0.76}\ (t/T)^{0.74}$ (when $t/T < 1$ use $t/T=1$)
Branch SIF Outplane, i_{ob}	$(0.038 + 2(d/D) + 2(d/D)^2 - 3.1(d/D)^3)\ (R/T)^{2/3}\ (t/T)$ (when $t/T < 1$ use $t/T=1$)
Branch SIF Torsional, i_{tb}	$0.45\ (R/T)^{0.8}\ (t/T)^{0.29}\ (d/D)^{2}$

Extruded outlet with $r_x \geq 0.05d$ $T < T_c < 1.5T$	(when r_x is not provided, use $r_x = 0$)
Term	**Equation**
Run Inplane Flexibility Factor, k_{ih}	$0.18 (R/T)^{0.91} (d/D)^5$
Run Outplane Flexibility Factor, k_{oh}	1
Run Torsional Flexibility Factor, k_{th}	$0.08 (R/T)^{0.91} (d/D)^{5.7}$
Branch Inplane Flexibility Factor, k_{ib}	$(1.91(d/D) – 4.32(d/D)^2 + 2.7(d/D)^3) (R/T)^{0.77} (d/D)^{0.47}(t/T)$
Branch Outplane Flexibility Factor, k_{ob}	$(0.34(d/D) – 0.49(d/D)^2 + 0.18(d/D)^3) (R/T)^{1.46} (t/T)$
Branch Torsional Flexibility Factor, k_{tb}	$(1.08(d/D) – 2.44(d/D)^2 + 1.52(d/D)^3) (R/T)^{0.77} (d/D)^{1.79}(t/T)$
Run SIF Inplane, i_{ih}	$(1+r_x /R)^{-2/3} (0.58(d/D)^{1.98}) (R/T)^{2/3}$
Run SIF Outplane, i_{oh}	$(1+r_x /R)^{-2/3} (0.49(d/D)^{2.69}) (R/T)^{2/3}$
Run SIF Torsional, i_{th}	$(1+r_x /R)^{-2/3} (0.58(d/D)^{2.69}) (R/T)^{2/3}$
Branch SIF Inplane i_{ib}	$(1+r_x /R)^{-2/3} (0.56(d/D)^{0.68}) (R/T)^{2/3}$ (when $t/T < 1$ use $t/T=1$)
Branch SIF Outplane, i_{ob}	$(1+r_x /R)^{-2/3} (0.71(d/D)^{0.5}) (R/T)^{2/3}$ (when $t/T < 1$ use $t/T=1$)
Branch SIF Torsional, i_{tb}	$(1+r_x /R)^{-2/3} (0.71(d/D)^2) (R/T)^{2/3}$

Welded-in contour insert	(when r_x is not provided, use $r_x = 0$)
Term	**Equation**
Run Inplane Flexibility Factor, k_{ih}	$0.18 (R/T)^{0.91} (d/D)^5$
Run Outplane Flexibility Factor, k_{oh}	1
Run Torsional Flexibility Factor, k_{th}	$0.1 (R/T)^{0.91} (d/D)^{5.7}$
Branch Inplane Flexibility Factor, k_{ib}	$(2.36(d/D) – 5.33(d/D)^2 + 3.33(d/D)^3) (R/T)^{0.77} (d/D)^{0.47} (t/T)$
Branch Outplane Flexibility Factor, k_{ob}	$(1+r_x /R) (0.67 (d/D) – 0.97(d/D)^2 + 0.36(d/D)^3) (R/T)^{1.46} (t/T)$
Branch Torsional Flexibility Factor, k_{tb}	$(1.05(d/D) – 2.36(d/D)^2 + 1.49(d/D)^3) (R/T)^{0.77} (d/D)^{1.61} (t/T)$
Run SIF Inplane, i_{ih}	$0.33 (R/T)^{2/3} (d/D)^{1.98}$
Run SIF Outplane, i_{oh}	$0.33 (R/T)^{2/3} (d/D)^{2.69}$
Run SIF Torsional, i_{th}	$0.33 (R/T)^{2/3} (d/D)^{2.69}$
Branch SIF Inplane i_{ib}	$0.37 (R/T)^{2/3} (d/D)^{0.86}$
Branch SIF Outplane, i_{ob}	$0.58 (R/T)^{2/3} (d/D)^{0.75}$
Branch SIF Torsional, i_{tb}	$0.37 (R/T)^{2/3} (d/D)^2$

Integrally Reinforced Forged Branch Outlet Fittings	
Term	**Equation**
Run Inplane Flexibility Factor, k_{ih}	$0.6 (R/T)^{0.5} (d/D)^5$
Run Outplane Flexibility Factor, k_{oh}	1
Run Torsional Flexibility Factor, k_{th}	$0.1 (R/T) (d/D)^{5.7}$
Branch Inplane Flexibility Factor, k_{ib}	$(0.55(d/D) – 1.13(d/D)^2 + 0.69(d/D)^3) (R/T) (t/T)$
Branch Outplane Flexibility Factor, k_{ob}	$(1.03(d/D) – 1.55(d/D)^2 + 0.59(d/D)^3) (R/T)^{1.5} (t/T) (d/D)^{0.33}$
Branch Torsional Flexibility Factor, k_{tb}	$(0.37(d/D) – 0.75(d/D)^2 + 0.46(d/D)^3) (R/T) (t/T) (d/D)^{1.2}$
Run SIF Inplane, i_{ih}	$(0.02 + 0.88(d/D) – 2.56(d/D)^2 + 2.58(d/D)^3) (R/T)^{0.43}$
Run SIF Outplane, i_{oh}	$(0.02 + 0.88(d/D) – 2.56(d/D)^2 + 2.58(d/D)^3) (R/T)^{0.43}$
Run SIF Torsional, i_{th}	$1.3 (R/T)^{0.45} (d/D)^{1.37}$
Branch SIF Inplane i_{ib}	$(0.08 + 1.28(d/D) – 2.35(d/D)^2 + 1.45(d/D)^3) (R/T)^{0.81} (t/T) (r/rp)$
Branch SIF Outplane, i_{ob}	$(1.83(d/D) – 1.07(d/D)^3) (R/T)^{0.82} (t/T) (r/rp)^{1.18}$
Branch SIF Torsional, i_{tb}	$0.77 (R/T)^{2/3} (t/T) (d/D)^2 (r/rp)$